In-Class Problems Week 11, Wed.

Problem 1.
Find the coefficients of

(a) \(x^5\) in \((1 + x)^{11}\)

(b) \(x^8 y^9\) in \((3x + 2y)^{17}\)

(c) \(a^6 b^6\) in \((a^2 + b^3)^5\)

Problem 2.
You want to choose a team of \(m\) people for your startup company from a pool of \(n\) applicants, and from these \(m\) people you want to choose \(k\) to be the team managers. You took 6.042, so you know you can do this in

\[
\binom{n}{m} \binom{m}{k}
\]

ways. But your CFO, who went to Harvard Business School, comes up with the formula

\[
\binom{n}{k} \binom{n - k}{m - k}.
\]

Before doing the reasonable thing—dump on your CFO or Harvard Business School—you decide to check his answer against yours.

(a) Give a combinatorial proof that your CFO’s formula agrees with yours.

(b) Verify this combinatorial proof by giving an algebraic proof of this same fact.

Problem 3. (a) Now give a combinatorial proof of the following, more interesting theorem:

\[
n2^{n-1} = \sum_{k=1}^{n} k \binom{n}{k}
\]

Hint: Let \(S\) be the set of all length-\(n\) sequences of 0’s, 1’s and a single *.

(b) Now prove (1) algebraically by applying the Binomial Theorem to \((1 + x)^n\) and taking derivatives.