In-Class Problems Week 8, Wed.

Problem 1. (a) Use the Pulverizer to find the inverse of 13 modulo 23 in the range \( \{1, \ldots, 22\} \).

(b) Use Fermat’s theorem to find the inverse of 13 modulo 23 in the range \( \{1, \ldots, 22\} \).

Problem 2. (a) Why is a number written in decimal evenly divisible by 9 if and only if the sum of its digits is a multiple of 9? Hint: \( 10 \equiv 1 \pmod{9} \).

(b) Take a big number, such as 37273761261. Sum the digits, where every other one is negated:

\[
3 + (-7) + 2 + (-7) + 3 + (-7) + 6 + (-1) + 2 + (-6) + 1 = -11
\]

Explain why the original number is a multiple of 11 if and only if this sum is a multiple of 11.

Problem 3.
The following properties of equivalence mod \( n \) follow directly from its definition and simple properties of divisibility. See if you can prove them without looking up the proofs in the text.

(a) If \( a \equiv b \pmod{n} \), then \( ac \equiv bc \pmod{n} \).

(b) If \( a \equiv b \pmod{n} \) and \( b \equiv c \pmod{n} \), then \( a \equiv c \pmod{n} \).

(c) If \( a \equiv b \pmod{n} \) and \( c \equiv d \pmod{n} \), then \( ac \equiv bd \pmod{n} \).

(d) \( \text{rem}(a,n) \equiv a \pmod{n} \).