State Machines

State machines

The state graph of a 99-bounded counter:

Start state

States: \{0, ..., 99, overflow\}

Transitions:

\[
\begin{align*}
0 &\rightarrow 1 \\
1 &\rightarrow 2 \\
2 &\rightarrow \cdots \\
99 &\rightarrow \text{overflow} \\
\text{overflow} &\rightarrow \text{overflow}
\end{align*}
\]

Die Hard

Simon says: On the fountain, there should be 2 jugs, do you see them? A 5-gallon and a 3-gallon. Fill one of the jugs with exactly 4 gallons of water and place it on the scale and the timer will stop. You must be precise; one ounce more or less will result in detonation. If you're still alive in 5 minutes, we'll speak.

Supplies:
- 3 Gallon Jug
- 5 Gallon Jug
Die Hard

Transferring water:

3 Gallon Jug 5 Gallon Jug

Die Hard state machine

State:
amount of water in jugs: \((b, l)\)
\[0 \leq b \leq 5, 0 \leq l \leq 3\]
Start State: \((0, 0)\)

State machines

5. Pour big jug into little jug
   (i) If ***no overflow***, then \((b, l) \rightarrow (0, b+l)\)
   \[b+l \leq 3\]
   (ii) otherwise \((b, l) \rightarrow (b-(3-l), 3)\)
6. Pour little jug into big jug.
   Likewise

State machines

Die Hard Transitions:
1. Fill little jug: \((b, l) \rightarrow (b, 3)\) for \(l < 3\)
2. Fill big jug: \((b, l) \rightarrow (5, l)\) for \(b < 5\)
3. Empty little jug: \((b, l) \rightarrow (b, 0)\) for \(l > 5\)
4. Empty big jug: \((b, l) \rightarrow (0, l)\) for \(b > 0\)

Simon’s challenge:
Disarm the bomb by putting precisely 4 gallons of water on the scale, or it will blow up.

(You can figure out how)
Die Hard once and for all

What if have a 9 gallon jug instead?

3 Gallon Jug 5 Gallon Jug 9 Gallon Jug

Can you do it? Can you prove it?

Preserved Invariants

Floyd's Invariant Method
(just like induction)

Base case: Show $P(\text{start})$

Preservation case: Show

if $P(q)$ and $q \rightarrow r$, then $P(r)$

Conclusion: $P$ holds for all reachable states, including final state (if any)

Die Hard Once & For All

Corollary: No state $(4,x)$ is reachable, so Bruce Dies!

The Diagonal Robot

it can move diagonally
The Diagonal Robot

can it get from (0,0) to (1,0)?

Robot Preserved Invariant

NO! preserved invariant:

\[ P((x, y)) ::= x + y \text{ is even} \]

move adds \( \pm 1 \) to both \( x \) & \( y \), preserving parity of \( x+y \). Also, \( P((0,0)) \) is true.

Robot Preserved Invariant

So all positions \((x,y)\) reachable from \((0,0)\) have \( x+y \) **even**. But \( 1 + 0 = 1 \) is odd, so \((1,0)\) is **not** reachable.

Robert W Floyd (1934–2001)

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Team Problems

Problems

1 & 2