Partial Orders & Scheduling

<table>
<thead>
<tr>
<th></th>
<th>4F.1</th>
<th>4F.2</th>
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Some Course 6 Prerequisites

<table>
<thead>
<tr>
<th>Course 6 Subject</th>
<th>Prerequisites</th>
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<tbody>
<tr>
<td>18.01</td>
<td>6.042</td>
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<tr>
<td>18.01</td>
<td>18.02</td>
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<tr>
<td>18.01</td>
<td>18.03</td>
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<td>6.001</td>
<td>6.034</td>
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<tr>
<td>6.042</td>
<td>6.046</td>
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<td>8.02</td>
<td>6.002</td>
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<td>18.03, 6.002</td>
<td>6.004</td>
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<td>6.033</td>
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<td>6.033</td>
<td>6.857</td>
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<tr>
<td>6.046</td>
<td>6.840</td>
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subjects with no prereq’s

<nothing> → d
“d is a Freshman subject”

minimal elements

\[
d \text{ is minimal: nothing else is smaller}
\]

\[
\forall c \neq d. \text{ NOT}(cRd)
\]

Minimal elements

\[
d \text{ is minimal: nothing else is smaller}
\]

\[
\forall c \neq d. \text{ NOT}(c \leq d)
\]

\[
d \text{ is minimum: smaller than everything else}
\]

\[
\forall c \neq d. \ d \leq c
\]

Constructing a Term Schedule

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identify minimal elements
Constructing a Term Schedule

Start schedule with them

Remove minimal elements

Identify new minimal elements

Schedule them next

Continue in this way...
**an antichain**

Set of subjects with no prereqs among them:
so can be taken in any order.
(said to be *incomparable*)

**some antichains**

many more...

**a chain**

Set of successive prereqs:
must be taken in order.
(subjects are *comparable*)

**some chains**

**maximum length chain**
how many terms to graduate?

5 terms are necessary to graduate -- because max chain length is 5 and 5 are sufficient -- if you can take unlimited subjects per term...

parallel processing time

min # terms to graduate:

\[ \text{min parallel time} = \text{max chain size}. \]

max term load:

\[ \text{\# processors for min time} \leq \text{max antichain size} \]

5 in this case

reduce the term load

max 4 subjects per term

3 Subjects per Term Possible
a leisurely schedule

Graduate taking only 1 subject/term?
Sure,

\[
\begin{array}{c}
18.01 & 6.002 & 8.02 & 6.003 & 18.03 & 6.004 & 6.043 & 18.02 & 6.004 \\
\end{array}
\]

\[
\begin{array}{c}
6.002 & 6.003 & 6.004 & 6.043 \\
\end{array}
\]

a topological sort

For min time: \( \geq 3 \)-subject term

13 subjects
max chain size = 5
so load of some term must be

\[
\geq \left\lfloor \frac{13}{5} \right\rfloor = 3
\]

Dilworth's Lemma

Prereq's among \( n \) subjects has

- a chain of size \( \geq t \) \( n \)
- or antichain of size \( \geq t \) \( n \)

for all \( 1 \leq t \leq n \).

Height/Birthday Partial Order

Two students are related to each other iff one is shorter and younger than the other.

\((s_1, a_1) \preceq (s_2, a_2)\)

iff \((s_1 \leq s_2) \) and \((a_1 \leq a_2)\)

(the product p.o.)

Dilworth Demo

Team Problems

Problems 1–3