Cardinality (the size of sets)

Surjective function from $A$ to $B$ implies $|A| \geq |B|$ for finite $A, B$

A bijection from $A$ to $B$ implies $|A| = |B|$
Same Size Infinite Sets?

\{1, 2, 3, 4, \ldots\}
and
\{0, 1, 2, 3, \ldots\}

a bijection

the “same size”!

size of the power set

\# subsets of a finite set \(A\)?

\(|\text{pow}(A)|\) ?

for \(A = \{a, b, c\}\), \(\text{pow}(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}\)

\text{pow}(A) \text{ bijection to bin-strings}

every computer scientist knows \#n-bit strings, so

Corollary:

\(|\text{pow}(A)| = 2^{|A|}\)

\text{pow}(N) \text{ bijection to 1 bit-strings}

infinite set \(N = \{0,1,2,\ldots\}\)

subset: \(\{0, 2, 3, \ldots\}\)

string: \(1\ 0\ 1\ 1\ 0\ 1\ \ldots\\)

a bijection from \(\text{pow}(N)\) to
infinite bit-strings, \(\{0,1\}^\omega\)
Team Problems

Problems

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