Convex vs. Non-Convex
Convex – only one minimum
Non-convex – multiple relative minima
\[ \min_x f(x) \]

Global Optimization (Deterministic)

![Figure 1. A function with relative minima.](image)

Convex Underestimator

![Figure 2. Convex underestimator.](image)

Choose a \( f(x) \), upper bound. Divide domain. Underestimate the lower bound with a parabola. Find minimum of parabola. Bound again. If new upper bound is lower than lower bound in other region, can stop considering that section.
To converge – lower bound rises at a certain rate; upper bound decreases at a certain rate.
Going several zones deep, creates many divisions: \(2^{N_{\text{divisions}}(1-D)}\)
Proteins: 100 dimensional space or more: \(100^N\) or more
Current papers: can solve 4-5 dimensions
Method guarantees global optimum (if you care about the global optimum)

If you have 20 variables, use heuristics that often find the global optimum, but there is no guarantee.

**Multi-start**

![Mesh of start points, weighted Monte Carlo](image)

**Figure 3.** Begin in multiple locations and then run minimization.

In low dimension, draw map. One method, do different starts. Run a local minimization on each, then compare values. With enough points, can make a space. Can use mesh. Can use Monte Carlo – random guess. If there are 100 points and 6 variables, \(100^6\) calculations.

**Simulated Annealing**

![Annealing process](image)

Can use when there are lots of global minimum

\[ f(x) \leftrightarrow kT \]

**Figure 4.** The molecule is heated and then cooled slowly so that conformational changes taking place will lead to a local minimum. This process is repeated many times until several closely related, low energy, conformations are obtained.

\[ f(x, z) \] mixed integer hybrid

**Genetic Algorithms**

\(\rightarrow\) discretize everything

“DNA” \((z_1, z_2, z_3, ...\)
mutate $z_n \rightarrow z'_n$

reproduction (exchange of DNA fragments)
replication
death
give everything probabilities to make it mirror evolution

Non-determinate methods $\Rightarrow$ do not exactly know when you are done