One more chemical subsystem
- diffusion + reaction problem
- processes occurring within/at cell membrane
  - cell migration
    - actin cytoskeleton
    - biological signals
    - cell membrane forward.
  - lipid bilayer is a barrier

Diffusion + rxn processes within/at plasma membrane.

Crucial point:
- actin polymerization & to how much the membrane has phosphate groups added on to it.

Top View of all

AKT activates the phosphorilation process by attacking itself on it.

more green, more AKT, more phosphates

Expt measurement:
- express Akt-GFP
- fluorescent green!!

Evanescent wave fluorescence microscopy
In the absence of any external ligand concentration gradient, cells generate a spatially-localized P lipid (Akt-GFP) distribution.

Internal symmetry breaking in the direction of the gradient.

Quantity of interest

\( x \)

# molecules
\( \text{P-lipids/\mu m}^2 \text{ cell membrane} \)

Geometry of the problem
Isotropic medium \( \Rightarrow \) no angular gradients.

Chemical process:
\( L + R \overset{V}{\rightleftharpoons} C \)
lipid receptor complex

lipid precision.
\( V \) velocity of rxn

\( P \)-lipid

\( K_c \) consumption
Bottom domain

mass cons. on 0-lipid.

\[ \frac{\partial x_b}{\partial t} = D \nabla_r^2 x_b - k_c x_b + v_b f(t) \]

\[ \nabla_r^2 = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) \quad 0 \leq r \leq R. \]

I.C. \( x_b(r, 0) = x_0 \)

B.C. \( r = 0, \quad \nabla_r x_b = 0 \)
\( r = R, \quad D \frac{\partial x_b}{\partial r} = g(t) \approx \text{flux from top domain} \)

Top domain

L-R binding dynamics

\[ \frac{\partial x_t}{\partial t} = D \nabla_\theta^2 x_t - k_c x_t + v_{ef}(t) \]

\[ \nabla_\theta^2 = \frac{1}{R^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) \]

Transform \( \theta \) coordinates to \( \zeta \) coordinates where \( \zeta = \cos \theta \)

\[ \nabla_\zeta^2 = \frac{1}{R^2 (1 - \zeta^2)^2} \frac{\partial^2}{\partial \zeta^2} \]

IC: \( x_t(\zeta, 0) = x_0 \)

BC: \( \zeta = 1, \quad \nabla_\zeta x_t = 0 \)
\( \zeta = 0, \quad \mathcal{R} \frac{\partial x_t}{\partial \zeta} \approx g(t) \approx \text{flux to bottom} \)

Is \( D_t = D_0 ? \)

\( V_t \neq V_b \) rxn velocity terms

good for lipids.

"integrin adhesion receptors"

- very active on the bottom that's adhesive to the substrate,
- very dynamic.
Scale variables
\[ \beta = \frac{\gamma}{\gamma}; \quad \lambda = \frac{\gamma}{\beta} \]

\[ \nabla^2 \phi = \frac{1}{\alpha^2} \nabla^2 \phi \]

**IC:** \( x_b(\rho,0) = x_0 \)

**BC:** \( \frac{\partial x_b}{\partial \rho} = 0; \quad \frac{\partial x_t}{\partial \rho} = 0, \quad \gamma = 1 \)

- Play with different geometries.
- You can solve (linear PDEs) by separation of variables or finite Fourier transform or Starville operator theory.

Details are in Haugh supplement on BE 430 website.

Result: \( x_b(\rho, \tau \to \infty) \), steady-state solution

(how long does it take to attain?)

To s:\space To ws?

\[ x_{bs}(\rho) = \rho + \frac{(V_t - V_b)}{2} \alpha \frac{I_0(\alpha \rho)}{I_1(\alpha \rho)} \]

\[ I_0, I_1 \text{ Bessel Functions.} \]

\[ p(\alpha) = \sum_{n=0}^{\infty} \frac{(4n+1)P_n(\alpha)}{1 + 2n(2n+1)\alpha^2} \]

Legendre Polynomial

\[ \alpha^2 = 3 \]

\[ D = 0.15 - \mu m^2/s \]

\[ k_c \sim 1 \text{ min}^{-1} \quad / \text{ To s, } \sim 30 \text{ sec.} \]

\[ R \sim 20 \mu m \]

**Notes:**
- \( V_t - V_b = \) hypothesis, predicted by model experimental test.